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Old-age mortality, the maximum life-span and their influence on variability of death and the rectangularization of the survival function

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Abstract

Rectangularization — the development of an increasingly rectangular shape of the survival curve — has been theoretically predicted and empirically shown among humans in nearly all low mortality countries in the last decade. Due to the differing pace of improvement of old-age mortality and the maximum life-span, rectangularization need to be approached from a different perspective. Explanations that account for this relation, however, remain limited. We propose a new approach which enables an investigation of the dependency of both issues. The Maximized Inner Rectangle Approach (MIRA), which determines the biggest rectangle under the survival curve, allows a decomposition of life years lived and lost, thereby enabling an examination of the used and unused human living potential. Different old-age mortality modeling strategies are applied and evaluated to model the maximum life-span and explore the relationship between the degree of rectangularization and the developments at the end of life.

1 Introduction

From a demographic point of view, the rise of life expectancy due to mortality declines at almost all ages is one of the most remarkable achievements. Expressed differently, rising life expectancy means improved survival. These changes reflect the rectangularization of the survival curve which is driven by an increasing number of people surviving to higher ages along with a decreasing variability of the age at death (cf. Cheung et al., 2005; Fries, 1980; Nusselder and Mackenbach, 1996; Wilmoth and Horiuchi, 1999).

Explanations that account for rectangularization are wide ranged (cf. Canudas-Romo, 2008; Cheung et al., 2005; Fries, 1980; Kannisto, 2000; Vaupel and Zhang, 2008; Wilmoth and Horiuchi, 1999; Yashin et al., 2002). Examples for such explanations are the Compression Hypothesis by Fries (1980), the Shifting Mortality Hypothesis by Bongaarts (2005) and Canudas-Romo (2008) and approaches concerning changing patterns as well as differing speed of improvements over the age range (cf. Vaupel and Zhang, 2008; Yashin et al., 2002). Fries (1980) defines a likely average age at death of 85 years with small variations around it and hypothesized a compression of mortality against this age. A shifting of mortality as well as varying age patterns and speed of improvements require at some point a flexible maximum life-span. In general, almost all of these concepts refer to a specific development of the upper limit. Hence, rectangularization towards an upper limit as well as variability patterns are closely related to the development of the maximum life-span.

Regrading this development, Wilmoth et al. (2000) state an increase since 1860s and further, show an acceleration of its around 1969 from 0.44 to 1.1 years per decade. This has implications for research on rectangularization as it extends current upper limits, towards rectangularization or shifting processes are proceeding. It simultaneously expands mortality and therefore, it is similarly effecting variability.

The aim of this study is to investigate these effects on rectangularization and variability. We are tackling this problem using a novel approach, named the maximizing inner rectangle approach (MIRA). MIRA enables a decomposition of longevity, measured as maximum life-span, into prematurely and longevity extended life-years lived and lost. Thereby, the incorporation of the maximum life-span is realized by assessing and testing different models, like the Kannisto-Model and other parametric models (cf. Thatcher et al., 1998). Moreover, simple approaches based on an assumed course of old and oldest-old mortality, like a constant course from a certain age onwards, are applied (Gampe, 2010). All applied approaches are carefully evaluated and validated to examine and prove their influence on estimated results. The estimations are based on mortality data from the Human Mortality Database (2013). This also allows the assessment of existing explanations and their further adjustment.

2 Maximized Inner Rectangle Approach

We introduce a novel tool to model and measure the influences of old-age mortality and the increase of the maximum life span on variability and survival, labeled the maximum inner rectangle approach (MIRA). The novel idea is based on the biggest possible rectangle, fitting to the area under the survival curve which is tangent to the curve in only one point. Therefore, this rectangle refers to an inner rectangularization, in contrast to the usual rectangularization towards some upper limit which we label outer rectangularization. However, both kinds are central dimensions in the illustration of old-age mortality influences on survival shapes.

The determination of the tangent point is based on some geometrical and formal demographic considerations. Typically, a rectangle is determined by two sides a and b. The area of a rectangle, A, is calculated by multiplying both sites. Hence, the area of any rectangle under the curve at age x, denoted by $A(x)$, is calculated by

$$A(x) = l(x) \cdot x. \tag{1}$$

The biggest rectangle under the curve demands a maximization of equation 1. The determination of the age maximizing the rectangle decomposes longevity, $l(0) \cdot \omega$ [1], into six areas which enable an examination of life years lived and lost due to premature death as well longevity extension. Figure 1 illustrates MIRA visually.

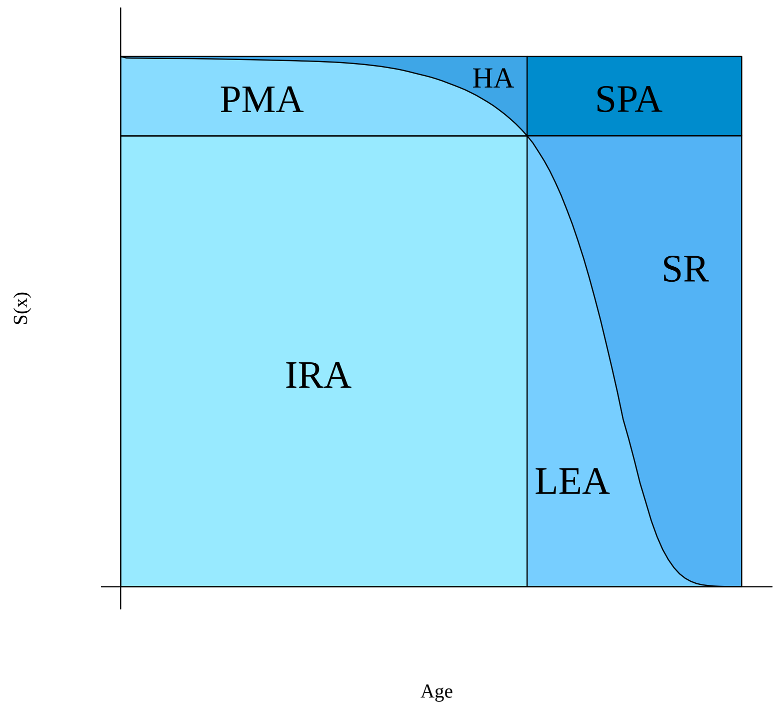

Figure 1: Visual Illustration of MIRA

(IRA – Inner Rectangle Area, PMA – Premature Mortality Area, LEA – Longevity Extension Area, HA – Horizontalization Area, SPA – Shifting Potential Area, SR – Senescence Rectangle)

3 Data and Methods

Parametric models are used to extrapolate the force of mortality at higher ages. This allows the empirical determination of a maximum age and, conducting an analysis over time, it also enables to some extend the analysis of old-age mortality improvements which, for instance, are necessary to shift the mode of death to higher ages (Canudas-Romo, 2010).

The choice of parametric models to model old and oldest-old mortality is based on previous findings of Thatcher et al. (1998). In this work, the Kannisto-Model, a logistic approach, proved as most suitable. Several other studies show that mortality decelerates at higher ages and even reaches a

[1] $\omega \sim$ the highest age attained

course of an approximately constant level (Gampe, 2010; Vaupel et al., 1998). Therefore and besides the classical parametric modeling strategies, we also use simple assumption based approaches, assuming, for instance, a constant chance of death of 0.5 from 110 onwards.
The empirical analysis is based on mortality and population data provided by the Human Mortality Database (2013).

4 Preliminary Results

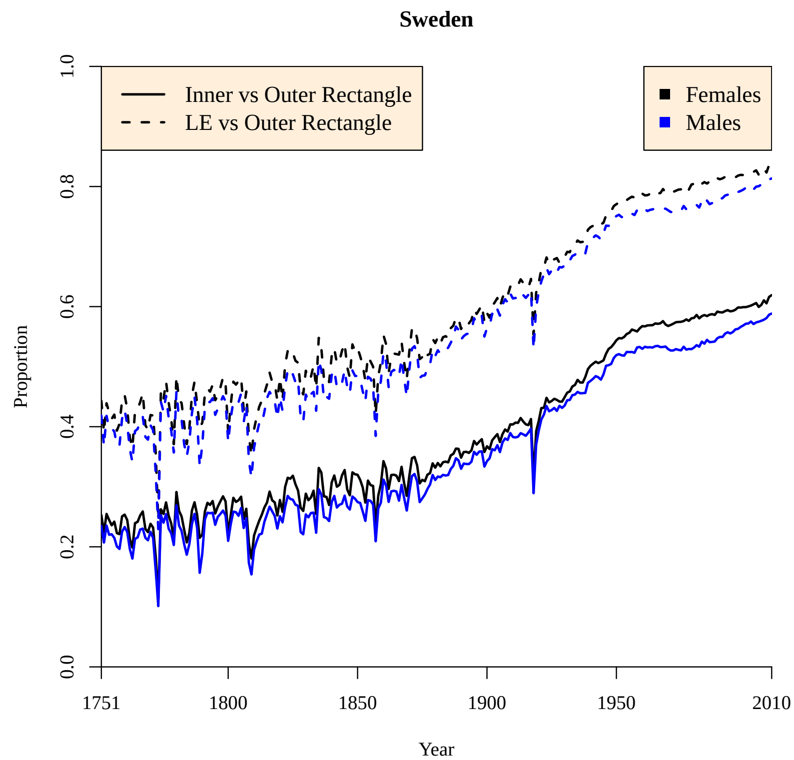

Figure 2: Preliminary Results of Inner vs. Outer Rectangularization, Sweden, 1751–2010

Figure 2 shows some preliminary basic analysis using data for Swedish females and males between 1751 and 2010. The solid line shows the proportion of the inner rectangle on the outer. Outer rectangularization, determined by longevity, is measured as moving rectangle with an upper border simply assumed to be the age where only 1% of the population is still alive (for the moving rectangle see: Wilmoth and Horiuchi, 1999). As mentioned earlier, the inner rectangle is the biggest rectangle fitting under the survival curve. The dashed line is the ratio between life expectancy at birth and the outer rectangle.
The course of the two ratios is approximately parallel. This suggests that the rectangular shape of the survival curve developed simultaneously to life expectancy without showing any signs of

compression against an upper which would require a convergence of both lines.
Regarding the comparatively low levels of premature mortality in Sweden since the 1960s, the development of old age mortality is a central force for the ongoing parallel development in the last 40 years. However, to what extend and what dynamics are responsible for this pattern remains unclear until recently.

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