Mortality Deceleration Revisited: Using the LAR Approach

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Abstract

The growing number of persons living beyond age 80 underscores the need for accurate measurement of mortality at advanced ages for population forecasting. One approach to study mortality patterns at advanced ages is based on Life table Aging Rate (LAR) after age 80. If mortality decelerates at older ages then the LAR values should decline with age rather than remain unchanged as in the case of Gompertz law. The LAR approach was applied to age-specific death rates for Canada, France, Sweden and the United States available in HMD. It was found that for all studied 24 single-year birth cohorts LAR does not change significantly with age in the age interval 80-100 years suggesting the validity of Gompertz law. Simulation study demonstrated that the apparent decline of LAR after age 80 found in earlier studies may be related to biased estimates of mortality rates measured in too wide 5-year age interval.

1. Introduction

Accurate estimates of mortality at advanced ages are essential for improving forecasts of mortality and predicting the population size of the oldest old age group. Earlier studies suggest that the exponential growth of mortality with age (Gompertz law) is followed by a period of deceleration, with slower rates of mortality increase (Greenwood and Irwin 1939; Horiuchi and Wilmoth 1998; Thatcher 1999; Thatcher, Kannisto and Vaupel 1998). It is believed that mortality at advanced ages has a tendency to deviate from the Gompertz law (Gavrilov and Gavrilova 1991), so that the logistic model is suggested for fitting human mortality after age 80 years (Horiuchi and Wilmoth 1998; Wilmoth et al. 2007).

Recent media reports (Financial Times, September 11, 2012; Wall Street Journal, March 2, 2012) revealed that official projections significantly overstated the number of centenarians both in the United States and the UK. Incorrect assumptions about mortality trajectories at advanced ages may be partially responsible for these projection inaccuracies.

In this study we analyze mortality trajectories at advanced ages with the life table aging rate approach using data on cohort mortality from the Human Mortality Database. Human Mortality Database became a traditional resource for demographers and actuaries. Life table aging rate (LAR) approach is often used in the study of mortality at older ages and was applied in the seminal paper devoted to the study of mortality deceleration (Horiuchi and Wilmoth 1998). We show that the choice of hazard rate estimator in LAR calculation may be crucial for conclusion about mortality trajectories at advanced ages. Comparison of different hazard rate estimators is made by computer simulation.

Analyzing mortality trajectories using life table aging rate

In 1990 Ansley Coale and Ellen Kisker proposed a method to calculate mortality schedules at advanced ages (Coale and Kisker 1990). This method is based on calculating a measure of mortality change that the authors called the age-specific rate of mortality change with age, or k_x. This measure is defined in the following way:

$$k_x = \ln(m_x) - \ln(m_{x-1})$$

where m_x is mortality rate at age x.

If k_x is a constant function of age then mortality follows the Gompertz law. If k_x declines with age then there is a mortality deceleration. Similar measure was also proposed by Horiuchi and Coale (Horiuchi and Coale 1990) and later this measure was called a life table aging rate (LAR) (Horiuchi and Wilmoth 1997).

This approach, based on calculating mortality change, was applied in several earlier studies of mortality trajectories at advanced ages (Horiuchi and Coale 1990; Horiuchi and Wilmoth 1998; Thatcher et al. 1998; Wilmoth 1995). These earlier studies demonstrated that values of k_x tend to decline after age 80 years suggesting mortality deceleration at advanced ages. Most of these studies used cross-sectional data. As we noted earlier, mortality deceleration may be caused by age misreporting at older ages and previous studies of mortality trajectories have been conducted almost twenty years ago. Thus, it is reasonable to repeat these earlier analyses using data on more recent extinct or almost extinct birth cohorts. In this paper we analyze mortality data for the following four countries: Canada, France, Sweden and the United States. Analyses are based on age-specific cohort death rates for the most recent extinct birth cohorts available in the Human Mortality Database (1894, 1896 and 1898 birth cohorts). Linear regression model was applied to verify if k_x is declining with age after age 80.

Figure 7 presents age pattern of k_x for Swedish male 1896 birth cohort. Note that values of k_x do not show any decline with age up to very advanced ages and after age 100 years random variation of k_x is very high. Overall, the k_x age pattern is in a good agreement with the Gompertz law and does not demonstrate any significant decline with age.

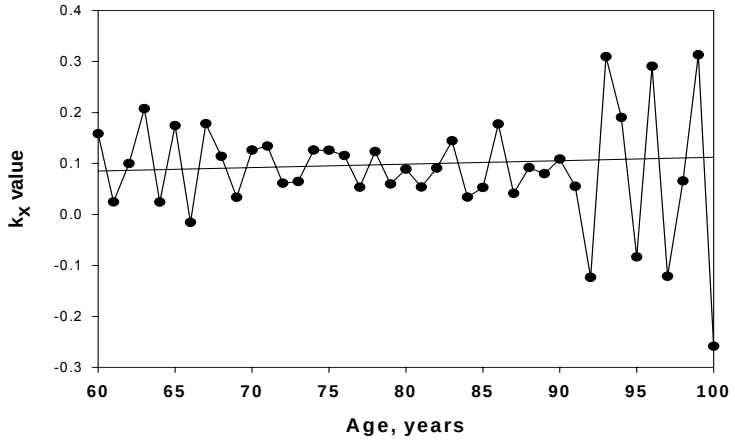

Figure 7. Age-specific rate of mortality change with age, k_x, Swedish males, 1896 birth cohort.

To quantify this finding, we conducted linear regression analyses for k_x as a dependent variable and age as a predictor variable in the age interval 80-100 years. If k_x does not change with age then the slope coefficient for this regression model should not be significantly different from zero. Table 2 presents regression slope coefficients and corresponding p-values for all 24 studied single-year birth cohorts.

As follows from Table 1, the slope coefficients for all studied birth cohorts are not significantly different from zero. Thus, the shape of mortality curve after age 80, as measured by the k_x function, appears to be consistent with the Gompertz model.

These results do not agree with earlier studies that showed linear decline of k_x after age 80 years (Horiuchi and Wilmoth 1998; Thatcher et al. 1998; Wilmoth 1995). There may be several reasons why these studies found mortality deceleration at advanced ages. In most cases these studies analyzed cross-sectional data combined into 5-year or 10-year time intervals, which may be prone to mortality deceleration due to secular decline of mortality.

Table 1. Slope parameter estimates and corresponding p-values for linear regression model of k_x as a function of age*, by country, sex and birth cohort.

Country	Sex	Birth cohort					
		1894		1896		1898	
		slope	p-value	slope	p-value	slope	p-value
Canada	F	-0.00023	0.914	0.00004	0.984	0.00066	0.583
	M	0.00112	0.778	0.00235	0.499	0.00109	0.678
France	F	-0.00070	0.681	-0.00179	0.169	-0.00165	0.181
	M	0.00035	0.907	-0.00048	0.808	0.00207	0.369
Sweden	F	0.00060	0.879	-0.00357	0.240	-0.00044	0.857
	M	0.00191	0.742	-0.00253	0.635	0.00165	0.792
USA	F	0.00016	0.884	0.00009	0.918	0.000006	0.994
	M	0.00006	0.965	0.00007	0.946	0.00048	0.610

* All regressions were run in the age interval 80-100 years.

Some studies analyzed cohort data aggregated into wide 5-year or 10-year birth cohorts. Such aggregation may result in spurious mortality deceleration if mortality in aggregated single-year birth cohorts is significantly different from each other. To test this hypothesis, we calculated k_x values using age-specific death rates for 5-year birth cohorts available in HMD. Table 2 shows slope parameters and corresponding p-values for linear regression model of k_x as a function of age in the age interval 80-100 years. Note that in the case of aggregated birth cohorts there are indeed some cohorts where k_x is declining with age.

Still only 4 out of 32 cohorts show negative slope coefficients suggesting decline in k_x with age and mortality deceleration. This example demonstrates that even in countries with smaller populations compared to the United States mortality deceleration at advanced ages is rather an exception than a rule. Thus, we may conclude that the analysis of age-specific rates of mortality change for four countries suggests that in most cases mortality deceleration at advanced ages is not supported by existing data. These results are mixed, because for some populations (French women) mortality deceleration does exist for all studied aggregated birth cohorts while in other countries (Canada) we do not observe mortality deceleration at all.

Table 2. Effect of birth cohort aggregation on mortality deceleration. Slope parameter estimates and corresponding p-values for linear regression model*, by country, sex and **5-year birth cohort**.

Country	Sex	Birth cohort							
		1880-84		1885-89		1890-94		1895-99	
		slope	p-value	slope	p-value	slope	p-value	slope	p-value
Canada	F	-0.00150	0.145	-0.00069	0.372	0.00015	0.851	-0.00002	0.983
	M	-0.00247	0.135	-0.00065	0.642	0.00094	0.306	0.00022	0.850
France	F	-0.00167	0.074	-0.00273	**0.047**	-0.00191	**0.005**	-0.00165	**0.002**
	M	-0.00072	0.818	-0.00082	0.515	-0.00049	0.661	-0.00047	0.412
Sweden	F	-0.00043	0.759	-0.00036	0.749	-0.00122	0.185	-0.00210	0.122
	M	0.00141	0.663	-0.00234	0.309	-0.00127	0.330	-0.00089	0.696
USA	F	-0.00131	0.113	-0.00030	0.654	-0.00027	0.685	0.00004	0.915
	M	-0.00187	**0.008**	-0.00050	0.417	-0.00039	0.399	0.00002	0.972

* All regressions were run in the age interval 80-100 years.

More important factor resulting in spurious decline of k_x is related to use of inappropriate measures of hazard rate at advanced ages. In the earlier cited studies of old-age mortality the values of k_x were calculated not for one-year but for 5-year age intervals:

$$k_x = \frac{\ln(m_x) - \ln(m_{x-5})}{5}$$

where m_x represents 5-year mortality rate.

It should be noted that 5-year age interval is very wide for analyzing mortality (and hazard rate) at advanced ages when mortality is particularly high. In this case the assumption about uniform distribution of deaths over the age interval (used for this estimate of hazard rate) does not work. As a result, hazard rate estimates become biased downward resulting in decline of 5-year k_x values with age. As an example of these effects, we present in Figure 8 the result of computer simulation using survival data, which follow the Gompertz model with typical parameters (see Appendix A for more detail). In this example we calculate mortality rates for 5-year age interval and then calculate age-specific mortality change function (k_x) using formula provided in (Wilmoth 1995). Theoretically we should expect to obtain constant value of k_x because our simulated data follow the Gompertz law. Instead we get declining pattern of k_x with age (see Figure 8), which is similar to trajectories reported in the previous publications (Horiuchi and Wilmoth 1998; Wilmoth 1995).

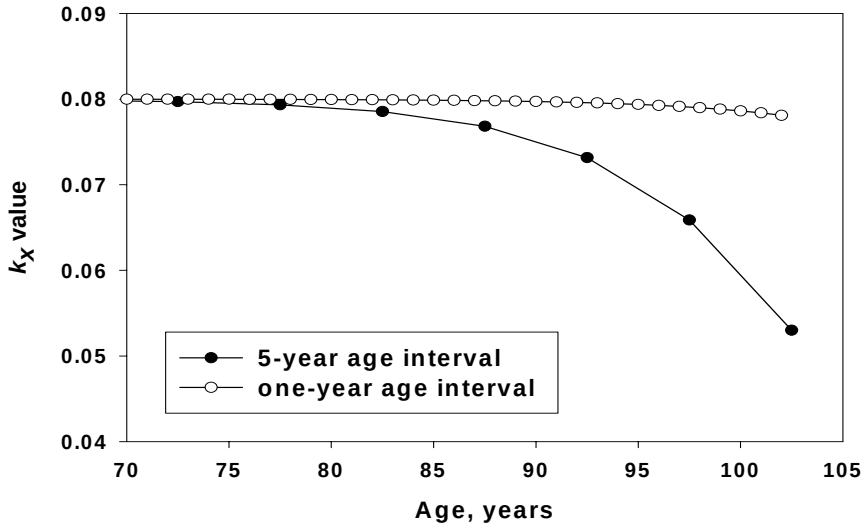

Figure 8. Age-specific rate of mortality change with age, k_x, by age interval for mortality calculation. Simulated data assuming that hazard rate follows the Gompertz law.

Thus, there is a possibility that the declining pattern of k_x with age found in earlier studies may be partially related to spurious mortality deceleration caused by using too wide age intervals for hazard rate estimates. It should be noted that in the first publication on this topic the authors used one-year smoothed estimates of mortality rates and found similar declining age pattern for k_x (Horiuchi and Coale 1990). There are two possible explanations for this phenomenon. One possibility is that the quality of age reporting at advanced ages for the studied populations was not sufficiently high for these time periods (1960s and 1970s). Another possibility is that the authors used cross-sectional data, which are more prone to demonstrate apparent mortality deceleration.

We may conclude that the analysis of age patterns of LAR for more recent single-year birth cohorts shows no evidence of mortality deceleration in the age interval 80-100 years.

4. Discussion

Study of age-specific rate of mortality change used as a measure of mortality deceleration found no mortality deceleration in the age interval 80-100 years for single-year birth cohorts of Canada, France, Sweden and the U.S. Thus, data suggest that mortality after age 80 follows the Gompertz model not only for the United States but also for countries with smaller population (like Sweden)

It should be noted that some researchers already found no mortality deceleration at advanced ages, but did not conduct a systematic study of this phenomenon. For example, Stauffer presents mortality of German cohorts, which shows no mortality deceleration up to age 90 years (Stauffer 2002). Other researchers who found no mortality deceleration at older ages for Canadian cohorts believed that this result is associated with problems of quality in their data (Bourbeau and Desjardins 2006). On the other hand, several systematic studies of mortality at older ages conducted in the 1990s came to a conclusion that mortality does decelerate after age 80 (Horiuchi and Wilmoth 1998; Thatcher 1999; Thatcher et al. 1998; Wilmoth 1995).

There are several reasons why earlier studies, including our own research (Gavrilov 1984; Gavrilov and Gavrilova 1991), reported mortality deceleration and mortality leveling-off at advanced ages (Horiuchi and Wilmoth 1998; Kannisto 1994; Robine and Vaupel 2001; Thatcher 1999; Thatcher et al. 1998). First, mortality deceleration may be caused by age misreporting in death data for older persons (Coale and Kisker 1986; Gavrilov and Gavrilova 2011). Earlier studies, conducted more than ten years

ago, used data for older birth cohorts when age reporting was not particularly accurate (Jdanov et al. 2008), even for such developed countries as the U.K (Gallop and Macdonald 2005).

Second, mortality deceleration may be a consequence of data aggregation. Most developed countries have much smaller populations compared to the United States and hence studies of mortality at advanced ages for these countries have to combine together many single-year birth cohorts thereby increasing the heterogeneity of the sample.

Finally, some researchers use inappropriate estimates of hazard rates when they study mortality at very high ages when hazard rate is high and changes rapidly. Many studies present information for age-specific probability of death rather than hazard rate (Gallop and Macdonald 2005; Gampe 2010; Modig, Drefahl and Ahlbom 2013; Robine and Vaupel 2001). It is not surprising that probability of death has a tendency of deceleration at advanced ages when mortality is high, taking into account that this mortality indicator has theoretical upper limit equal to one (see Appendix A). For example, a study of mortality among supercentenarians demonstrated that probability of death for this group does not increase with age (Robine and Vaupel 2001). Some authors do not distinguish between probability of death and hazard rate in their calculations (Le Bras 2005).

Mortality rates calculated for wide age intervals also produce biased estimates of hazard rates. For example, use of five-year mortality rates may produce a spurious evidence of mortality deceleration when age-specific rate of mortality change is analyzed (see previous section). Loss of individuals to follow-up in longitudinal study may also be a factor contributing to apparent mortality deceleration at advanced ages (Manton, Akushevich and Kulminski 2008). Appendix A compares accuracy of various estimates of hazard rate at advanced ages and provides a degree of deviation from the correct theoretical values of hazard rate. It appears that age misreporting, use of inappropriate estimates of hazard rates and perhaps data heterogeneity could lead to downward biases in mortality estimates at older ages reported in previous studies.

The results obtained in this study may be important for mortality forecasting, particularly if mortality is predicted for birth cohorts. These results also may be significant for projections of the size of older population. As we already noted, inappropriate assumptions about mortality at advanced ages may be partially responsible for these projection inaccuracies of older population in the United States and the United Kingdom. .

5. Conclusion

Few people survive to advanced ages and, in standard mortality tables, it is frequently necessary to compile data over an entire decade to obtain a sufficiently large sample. Our work shows that the observed deceleration in measured mortality rates could result in part from the heterogeneity of the data. The second problem we examined is frequently overlooked by demographers and actuaries: the problem of correct estimation of the instantaneous mortality rate (hazard rate). At the most advanced ages, the rates of death are so high that it is impossible to assume that the number of dying is distributed uniformly within the studied one-year age intervals. As a result, the estimates of mortality rates (or central death rates) are biased downward at advanced ages. And finally, the third problem is related to the fact that elderly people tend to exaggerate their age. In the United States, this may have impaired the accuracy of mortality rate estimates in the past.

Appendix A.
Hazard rate (mortality force) estimation at advanced ages: A simulation study

A conventional way to obtain estimates of mortality at advanced ages is a construction of demographic life table with probability of death (q_x) as one of important life table functions. Although probability of death is a useful indicator for mortality studies, it may not be the most convenient one for studies of mortality at advanced ages. First, the values of q_x depend on the length of the age interval Δx for which it is calculated. This hampers both analyses and interpretation. Also, by definition q_x is bounded by

unity, which would inevitably produce apparent mortality deceleration when death rates are particularly high.

More useful indicator of mortality at advanced age is instantaneous mortality rate (mortality force) or hazard rate, μ_x which is defined as follows:

$$\mu_x = -\frac{dN_x}{N_x dx} = -\frac{d\ln(N_x)}{dx} \approx -\frac{\Delta\ln(N_x)}{\Delta x}$$

where N_x is a number of living individuals exposed to risk of death at age x. It follows from the definition of hazard rate that it is equal to the rate of decrease of logarithmic survival function with age. In actuarial practice, hazard rate is often called mortality force as it was done in the original paper by Benjamin Gompertz (Gompertz 1825). Hazard rate does not depend on the length of the age interval (it is measured at the instant of time x), has no upper boundary and has a dimension of rate (time^{-1}). It should also be noted that the famous law of mortality, the Gompertz law, was first proposed for fitting the age-specific hazard rate function rather than probability of death (Gompertz 1825).

The empirical estimates of hazard rates are often based on suggestion that age-specific mortality rate or death rate (number of deaths divided by exposure) is a good estimate of theoretical hazard rate. One of the first empirical estimates of hazard rate was proposed by George Sacher (Sacher 1956; Sacher 1966):

$$\mu_x = \frac{1}{\Delta x}[\ln(l_{x-\frac{\Delta x}{2}}) - \ln(l_{x+\frac{\Delta x}{2}})] = \frac{1}{\Delta x}\ln\left(\frac{l_{x-\frac{\Delta x}{2}}}{l_{x+\frac{\Delta x}{2}}}\right)$$

This estimate is unbiased for slow changes in hazard rate if $\Delta x \Delta \mu_x << 1$ (Sacher, 1966) and was shown to be the maximum likelihood estimate (Gehan and Siddiqui 1973). A simplified version of Sacher estimate (for small age intervals equal to unity) is often used in demographic studies of mortality: $\mu_x = -\ln(1-q_x)$. This estimate was initially suggested by Gehan who called it a 'Sacher' estimate (Gehan 1969; Gehan and Siddiqui 1973). It is based on the assumption that hazard rate is constant over age interval and is shifted by one half of a year to younger ages compared to the original Sacher estimate.

Another empirical estimate of hazard rate, often used in life table construction (Klein and Moesberger 1997), is the actuarial estimate, which is calculated in the following way (Kimball 1960):

$$\mu_x = \frac{2q_x}{\Delta x\,(2-q_x)} = \frac{2}{\Delta x}\,\frac{l_{x-\Delta x} - l_x}{l_{x-\Delta x} + l_x}$$

This estimate assumes uniform distribution of deaths over the age interval and is bounded by $2/\Delta x$, so this is not the best estimate of hazard rate at extreme old ages when death rates are particularly high (Gavrilov and Gavrilova 1991).

At advanced ages, when death rates are very high, the assumptions about small changes in hazard rate or a constant hazard rate within the age interval become questionable. The same is true for the assumption of uniform distribution of deaths within the age interval.

We conducted a simulation study in order to compare and evaluate the accuracy of different empirical estimates of hazard rate. For this purpose values of survivors at each age were calculated

assuming that age-specific hazard rate follows the Gompertz law. The theoretical equation was drawn by integrating the Gompertz formula (Gavrilov, Gavrilova and Nosov 1983):

$$\frac{N_x}{N_0} = \frac{N_{x0}}{N_0}\exp\left[\left(-\frac{a}{b}\right)(e^{b\,x} - e^{b\,x0})\right]$$

where N_x/N_0 is the probability of survival to age x, i.e. the number of hypothetical cohort at age x divided by its initial number N_0. *a* and *b* are the parameters of Gompertz equation (see formula 1). The simulation assumed that the Gompertz law works for the entire age interval and the initial cohort size is equal to 10^{11} individuals. The Gompertz parameters are typical for the U.S. birth cohorts: slope coefficient (*b*) = 0.08 $year^{-1}$; *a*= 0.0001 $year^{-1}$. The main focus of this study was on older ages beyond 90 years. Accuracy of various hazard rate estimates (Sacher, Gehan, and actuarial estimates) and probability of death is compared at ages 100 and 110 years.

Figure 9 shows theoretical and empirically estimated values of hazard rate after age 90 using the Sacher estimate of hazard rate and one-year probability of death. Note that the Sacher estimates practically coincide with theoretical mortality trajectory. At the same time, probability of death strongly underestimates mortality after age 100.

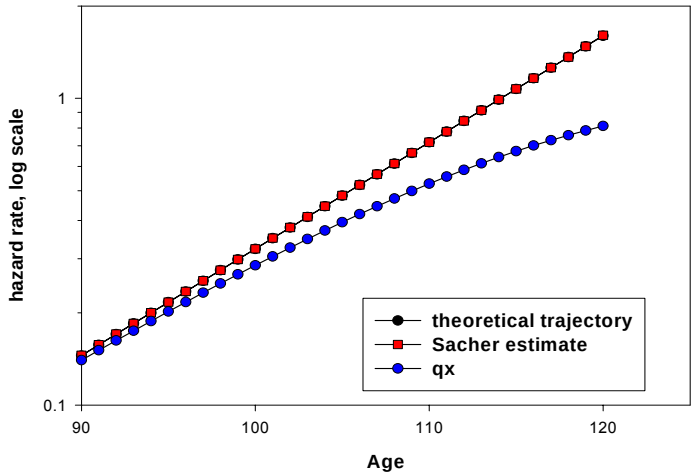

Figure 9. Comparison of Sacher estimate of hazard rate with one-year probability of death using simulated data based on the Gompertz mortality model.

Figure 10 compares theoretical values of hazard rate with empirical estimates of hazard rate using actuarial estimate of hazard rate (equivalent to the age-specific death rate or mortality rate, m_x). The actuarial estimates underestimate mortality at later age (110 years) compared to one-year probability of death.

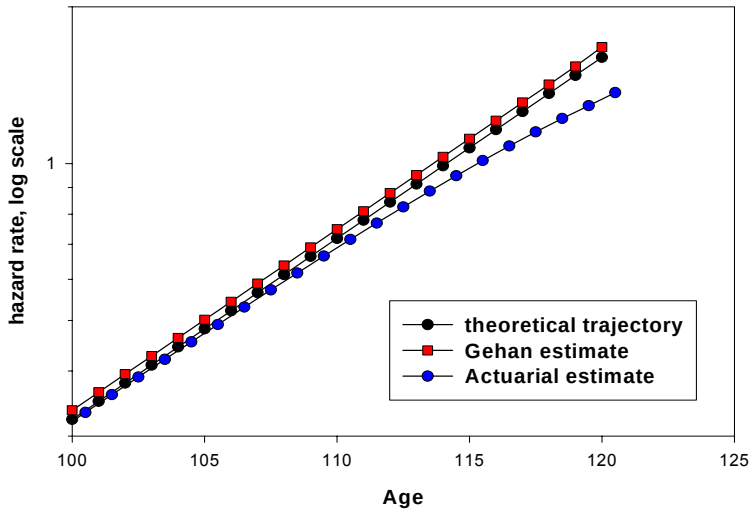

Figure 10. Comparison of actuarial estimate of hazard rate with Gehan estimate of hazard rate using simulated data based on the Gompertz mortality model.

Table 3 summarizes the results of our simulation study. It demonstrates that the Sacher estimate of hazard rate is the best one for use at advanced ages. These results underscore that the choice of proper hazard rate estimate is of paramount importance when mortality trajectory at advanced ages is analyzed. Sacher estimate turned out to be the most accurate estimate for advanced ages while one-year probability of death deviates from hazard rate function after age 85 years. Unfortunately, standard statistical packages do not use the Sacher estimate in their calculations of hazard rate.

Table 3. Comparison of different estimates of hazard rate with theoretical simulated values of hazard rate based on the Gompertz model.

Estimate of hazard rate	**Hazard rate estimate at age 100 years**	**Hazard rate estimate at age 110 years**
Probability of death	11.6% understatement	26.7% understatement
Sacher estimate	0.1% overstatement	0.1% overstatement
Actuarial estimate	1.0% understatement	4.5% understatement

Some statistical packages may produce biased estimates of hazard rates at advanced ages. This is the case for the Nelson-Aalen hazard rate estimates provided by sts command of Stata statistical software (StataCorp 2009). In fact, the Nelson-Aalen method was initially proposed for cumulative hazard rate estimation (particularly for right-censored survival data) (Klein and Moesberger 1997). In Stata, hazard rate estimation is made by taking the steps of the Nelson-Aalen cumulative hazard function (Cleves et al. 2008), so that for each observed time of death, x_j the estimated hazard contribution is:

$$\Delta\hat{H}(x_j) = \hat{H}(x_j) - \hat{H}(x_{j-1})$$

where $\hat{H}(x)$ is an estimate of cumulative hazard function.

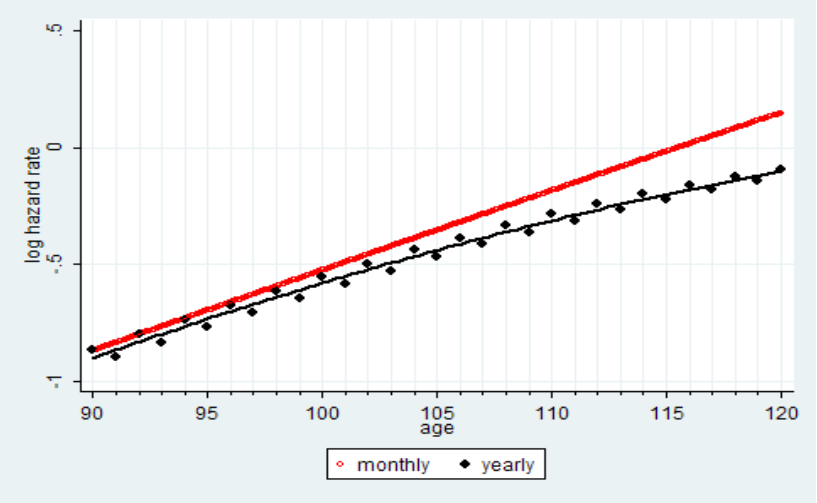

Figure 11. Comparison of monthly and yearly Nelson-Aalen estimates of hazard rate using simulated mortality data based on the Gompertz model

The way of hazard rate estimation conducted in Stata is similar to calculation of life table probability of death (StataCorp 2009), i.e. the number of deaths in the studied age interval is divided by the number alive at the beginning of age interval. At advanced ages when mortality is high and for relatively wide age intervals, number of persons exposed to risk of death in the middle of age interval is substantially lower than the number alive at the beginning of age interval. This would result in downward bias in hazard rate estimates at advanced ages, which is observed when the Nelson-Aalen estimates are applied to yearly age intervals. Simulation studies showed that the bias in hazard rate estimation increases with the increase of the age interval (Kimball 1960). Narrowing the age interval for hazard rate estimation from one year to one month helps to improve the accuracy of hazard rate estimation. For smaller monthly age intervals, the problem described above is not so crucial and the Nelson-Aalen method still can be applied. Figure 11 shows Nelson-Aalen hazard rate estimates produced by sts command of Stata for yearly and monthly age intervals using our simulated Gompertz mortality data. Note that hazard rates estimated for yearly age intervals demonstrate substantial mortality deceleration while hazard rate estimates calculated for monthly age intervals follow the Gompertz model.

Mortality deceleration and even mortality decline at advanced ages may occur when hazard rates are being smoothed using kernel smooth procedure. Figure 12 shows mortality trajectory at advanced ages when Stata kernel smoothing procedure (with default settings) is applied to our simulated Gompertz mortality data.

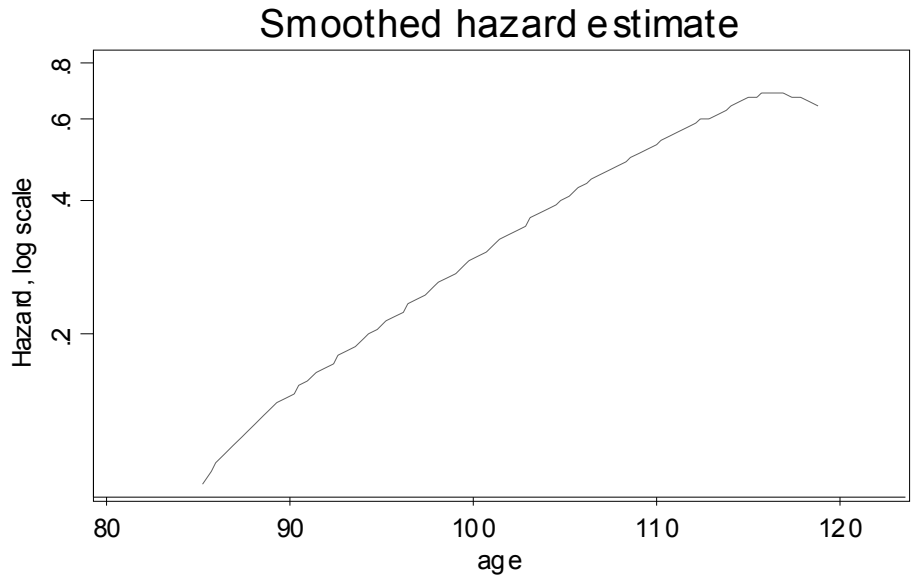

Figure 12. Simulated mortality data based on the Gompertz model after the kernel smooth procedure provided by the Stata statistical package.

Smoothing procedures assume mortality averaging over rather wide age interval (bandwidth), which leads to mortality underestimations at very advanced ages when hazard rates grow very rapidly.

These examples suggest that even standard estimates of hazard rates provided by statistical packages should be treated with caution when mortality is studied at very advanced ages.

Acknowledgements

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